

Overview of Statistical Methods

Prepare

1. Context

What do the data mean?

What is the goal of study?

2. Source of the Data

Are the data from a source with a special interest so that there is pressure to obtain results that are favorable to the source?

3. Sampling Method

Were the data collected in a way that is unbiased, or were the data collected in a way that is biased (such as a procedure in which respondents volunteer to participate)?

Analyze

1. Graph the Data

2. Explore the Data

Are there any outliers (numbers very far away from almost all of the other data)?

What important statistics summarize the data (such as the mean and standard deviation)?

How are the data distributed?

Are there missing data?

Did many selected subjects refuse to respond?

3. Apply Statistical Methods

Use technology to obtain results

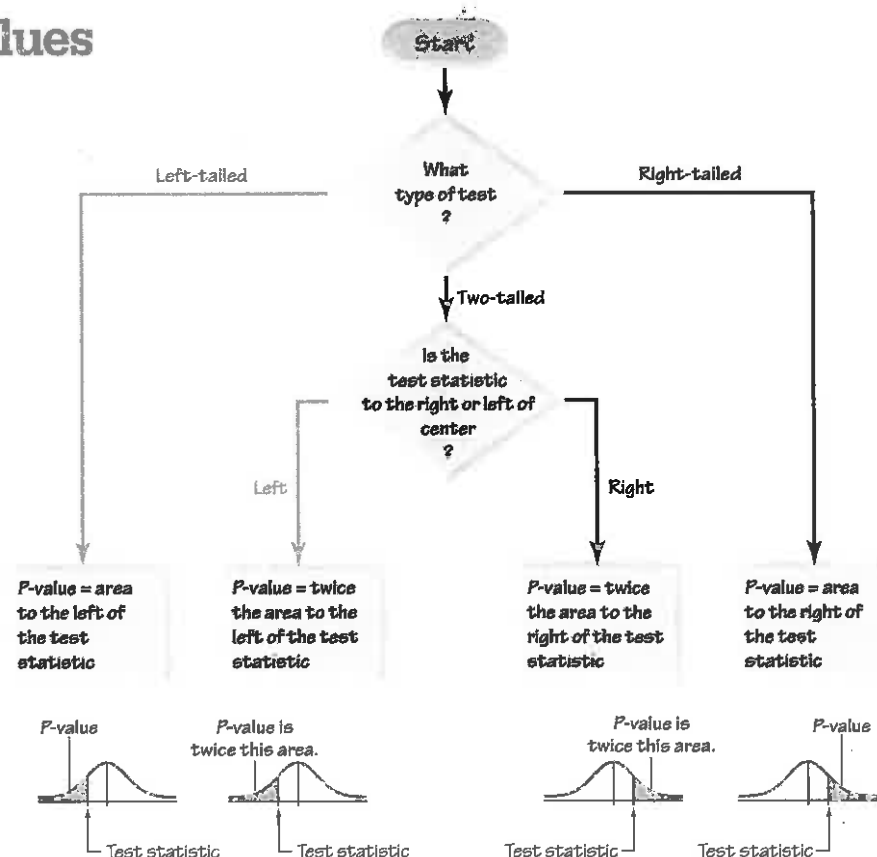
Conclude

1. Statistical Significance

Do the results have statistical significance?

Do the results have practical significance?

Finding P-Values



Inferences about μ : choosing between t and normal distributions

t distribution: or σ not known and normally distributed population
 σ not known and $n > 30$

Normal distribution: or σ known and normally distributed population
 σ known and $n > 30$

Nonparametric method or bootstrapping: Population not normally distributed and $n \leq 30$

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Ch. 3: Descriptive Statistics $\bar{x} = \frac{\sum x}{n} \quad \text{Mean}$ $\bar{x} = \frac{\sum (f \cdot x)}{\sum f} \quad \text{Mean (frequency table)}$ $s = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}} \quad \text{Standard deviation}$ $s = \sqrt{\frac{n(\sum x^2) - (\sum x)^2}{n(n - 1)}} \quad \text{Standard deviation (shortcut)}$ $s = \sqrt{\frac{n[\sum (f \cdot x^2)] - [\sum (f \cdot x)]^2}{n(n - 1)}} \quad \text{Standard deviation (frequency table)}$ $\text{variance} = s^2$	Ch. 7: Confidence Intervals (one population) $\hat{p} - E < p < \hat{p} + E \quad \text{Proportion}$ where $E = z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}}$ $\bar{x} - E < \mu < \bar{x} + E \quad \text{Mean}$ where $E = t_{\alpha/2} \frac{s}{\sqrt{n}}$ (σ unknown) or $E = z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$ (σ known) $\frac{(n - 1)s^2}{\chi_R^2} < \sigma^2 < \frac{(n - 1)s^2}{\chi_L^2} \quad \text{Variance}$
Ch. 4: Probability $P(A \text{ or } B) = P(A) + P(B)$ if A, B are mutually exclusive $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ if A, B are not mutually exclusive $P(A \text{ and } B) = P(A) \cdot P(B)$ if A, B are independent $P(A \text{ and } B) = P(A) \cdot P(B A)$ if A, B are dependent $P(A) = 1 - P(\bar{A})$ Rule of complements ${}_nP_r = \frac{n!}{(n - r)!} \quad \text{Permutations (no elements alike)}$ $\frac{n!}{n_1! n_2! \cdots n_k!} \quad \text{Permutations (} n_1 \text{ alike, } \dots \text{)}$ ${}_nC_r = \frac{n!}{(n - r)! r!} \quad \text{Combinations}$	Ch. 7: Sample Size Determination $n = \frac{[z_{\alpha/2}]^2 0.25}{E^2} \quad \text{Proportion}$ $n = \frac{[z_{\alpha/2}]^2 \hat{p}\hat{q}}{E^2} \quad \text{Proportion } (\hat{p} \text{ and } \hat{q} \text{ are known)}$ $n = \left[\frac{z_{\alpha/2} \sigma}{E} \right]^2 \quad \text{Mean}$
Ch. 5: Probability Distributions $\mu = \sum [x \cdot P(x)] \quad \text{Mean (prob. dist.)}$ $\sigma = \sqrt{\sum [x^2 \cdot P(x)] - \mu^2} \quad \text{Standard deviation (prob. dist.)}$ $P(x) = \frac{n!}{(n - x)! x!} \cdot p^x \cdot q^{n-x} \quad \text{Binomial probability}$ $\mu = n \cdot p \quad \text{Mean (binomial)}$ $\sigma^2 = n \cdot p \cdot q \quad \text{Variance (binomial)}$ $\sigma = \sqrt{n \cdot p \cdot q} \quad \text{Standard deviation (binomial)}$ $P(x) = \frac{\mu^x \cdot e^{-\mu}}{x!} \quad \text{Poisson distribution}$ where $e = 2.71828$	Ch. 9: Confidence Intervals (two populations) $(\hat{p}_1 - \hat{p}_2) - E < (p_1 - p_2) < (\hat{p}_1 - \hat{p}_2) + E$ where $E = z_{\alpha/2} \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}}$ $(\bar{x}_1 - \bar{x}_2) - E < (\mu_1 - \mu_2) < (\bar{x}_1 - \bar{x}_2) + E \quad (\text{Indep.})$ where $E = t_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$ (df = smaller of $n_1 - 1, n_2 - 1$) (σ_1 and σ_2 unknown and not assumed equal) $E = t_{\alpha/2} \sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}} \quad (\text{df} = n_1 + n_2 - 2)$ $s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)}$ (σ_1 and σ_2 unknown but assumed equal) $E = z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$ (σ_1, σ_2 known)
Ch. 6: Normal Distribution $z = \frac{x - \mu}{\sigma} \text{ or } \frac{x - \bar{x}}{s} \quad \text{Standard score}$ $\mu_{\bar{x}} = \mu \quad \text{Central limit theorem}$ $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \quad \text{Central limit theorem (Standard error)}$	$\bar{d} - E < \mu_d < \bar{d} + E \quad (\text{Matched pairs})$ where $E = t_{\alpha/2} \frac{s_d}{\sqrt{n}}$ (df = $n - 1$)

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Ch. 8: Tests for one population $z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$ Proportion—one population $t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$ Mean—one population (σ unknown) $z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$ Mean—one population (σ known) $\chi^2 = \frac{(n-1)s^2}{\sigma^2}$ Standard deviation or variance—one population	Ch. 10: Linear Correlation/Regression Correlation $r = \frac{n\sum xy - (\sum x)(\sum y)}{\sqrt{n(\sum x^2) - (\sum x)^2} \sqrt{n(\sum y^2) - (\sum y)^2}}$ or $r = \frac{\sum (z_x z_y)}{n-1}$ where $z_x = z$ score for x $z_y = z$ score for y Slope: $b_1 = \frac{n\sum xy - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$ or $b_1 = r \frac{s_y}{s_x}$ y-Intercept: $b_0 = \bar{y} - b_1 \bar{x}$ or $b_0 = \frac{(\sum y)(\sum x^2) - (\sum x)(\sum xy)}{n(\sum x^2) - (\sum x)^2}$ $\hat{y} = b_0 + b_1 x$ Estimated eq. of regression line $r^2 = \frac{\text{explained variation}}{\text{total variation}}$ $s_e = \sqrt{\frac{\sum (y - \hat{y})^2}{n-2}}$ or $\sqrt{\frac{\sum y^2 - b_0 \sum y - b_1 \sum xy}{n-2}}$ $\hat{y} - E < y < \hat{y} + E$ Prediction interval where $E = t_{\alpha/2, s_e} \sqrt{1 + \frac{1}{n} + \frac{n(x_0 - \bar{x})^2}{n(\sum x^2) - (\sum x)^2}}$
Ch. 9: Test Statistics: Two populations $z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\hat{p}\hat{q}}{n_1} + \frac{\hat{p}\hat{q}}{n_2}}}$ Two proportions $\leftarrow \bar{p} = \frac{x_1 + x_2}{n_1 + n_2}$ $t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$ df = smaller of $n_1 - 1, n_2 - 1$ Two means—-independent; σ_1 and σ_2 unknown, and not assumed equal. $t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}}$ (df = $n_1 + n_2 - 2$) $s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$ Two means—-independent; σ_1 and σ_2 unknown, but assumed equal. $z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$ Two means—-independent; σ_1, σ_2 known. $t = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}}$ Two means—matched pairs (df = $n - 1$) $F = \frac{s_1^2}{s_2^2}$ Standard deviation or variance—two populations (where $s_1^2 \geq s_2^2$)	Ch. 12: One-Way Analysis of Variance Procedure for testing $H_0: \mu_1 = \mu_2 = \mu_3 = \dots$ 1. Use software or calculator to obtain results. 2. Identify the P-value. 3. Form conclusion: If $P\text{-value} \leq \alpha$, reject the null hypothesis of equal means. If $P\text{-value} > \alpha$, fail to reject the null hypothesis of equal means.
Ch. 11: Goodness-of-Fit and Contingency Tables $\chi^2 = \sum \frac{(O - E)^2}{E}$ Goodness-of-fit (df = $k - 1$) $\chi^2 = \sum \frac{(O - E)^2}{E}$ Contingency table [df = $(r - 1)(c - 1)$] where $E = \frac{(\text{row total})(\text{column total})}{(\text{grand total})}$ $\chi^2 = \frac{(b - c - 1)^2}{b + c}$ McNemar's test for matched pairs (df = 1)	Ch. 12: Two-Way Analysis of Variance Procedure: 1. Use software or a calculator to obtain results. 2. Test H_0: There is no interaction between the row factor and column factor. 3. Stop if H_0 from Step 2 is rejected. If H_0 from Step 2 is not rejected (so there does not appear to be an interaction effect), proceed with these two tests: Test for effects from the row factor. Test for effects from the column factor.

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Ch. 13: Nonparametric Tests

$$z = \frac{(x + 0.5) - (n/2)}{\frac{\sqrt{n}}{2}} \quad \text{Sign test for } n > 25$$

$$z = \frac{T - n(n+1)/4}{\sqrt{\frac{n(n+1)(2n+1)}{24}}} \quad \text{Wilcoxon signed ranks (matched pairs and } n > 30)$$

$$z = \frac{R - \mu_R}{\sigma_R} = \frac{R - \frac{n_1(n_1 + n_2 + 1)}{2}}{\sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}}} \quad \text{Wilcoxon rank-sum (two independent samples)}$$

$$H = \frac{12}{N(N+1)} \left(\frac{R_1^2}{n_1} + \frac{R_2^2}{n_2} + \dots + \frac{R_k^2}{n_k} \right) - 3(N+1)$$

Kruskal-Wallis (chi-square df = $k - 1$)

$$r_s = 1 - \frac{6 \sum d^2}{n(n^2 - 1)} \quad \text{Rank correlation}$$

$$\left(\text{critical values for } n > 30: \frac{\pm z}{\sqrt{n-1}} \right)$$

$$z = \frac{G - \mu_G}{\sigma_G} = \frac{G - \left(\frac{2n_1 n_2}{n_1 + n_2} + 1 \right)}{\sqrt{\frac{(2n_1 n_2)(2n_1 n_2 - n_1 - n_2)}{(n_1 + n_2)^2 (n_1 + n_2 - 1)}}} \quad \text{Runs test for } n > 20$$

Ch. 14: Control Charts

R chart: Plot sample ranges

$$\text{UCL: } D_4 \bar{R}$$

$$\text{Centerline: } \bar{R}$$

$$\text{LCL: } D_3 \bar{R}$$

$\bar{x}$ chart: Plot sample means

$$\text{UCL: } \bar{\bar{x}} + A_2 \bar{R}$$

$$\text{Centerline: } \bar{\bar{x}}$$

$$\text{LCL: } \bar{\bar{x}} - A_2 \bar{R}$$

p chart: Plot sample proportions.

$$\text{UCL: } \bar{p} + 3\sqrt{\frac{\bar{p}\bar{q}}{n}}$$

$$\text{Centerline: } \bar{p}$$

$$\text{LCL: } \bar{p} - 3\sqrt{\frac{\bar{p}\bar{q}}{n}}$$

Table A-6 Critical Values of the Pearson Correlation Coefficient r

n	$\alpha = .05$	$\alpha = .01$
4	.950	.990
5	.878	.959
6	.811	.917
7	.754	.875
8	.707	.834
9	.666	.798
10	.632	.765
11	.602	.735
12	.576	.708
13	.553	.684
14	.532	.661
15	.514	.641
16	.497	.623
17	.482	.606
18	.468	.590
19	.456	.575
20	.444	.561
25	.396	.505
30	.361	.463
35	.335	.430
40	.312	.402
45	.294	.378
50	.279	.361
60	.254	.330
70	.236	.305
80	.220	.286
90	.207	.269
100	.196	.256

NOTE: To test $H_0: \rho = 0$ against $H_1: \rho \neq 0$, reject H_0 if the absolute value of r is greater than the critical value in the table.

Control Chart Constants

Subgroup Size n	D_3	D_4	A_2
2	0.000	3.267	1.880
3	0.000	2.574	1.023
4	0.000	2.282	0.729
5	0.000	2.114	0.577
6	0.000	2.004	0.483
7	0.076	1.924	0.419

NEGATIVE z Scores

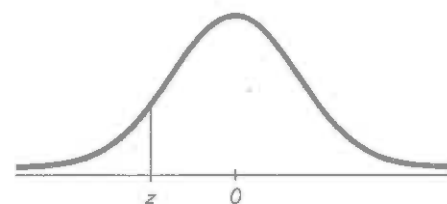


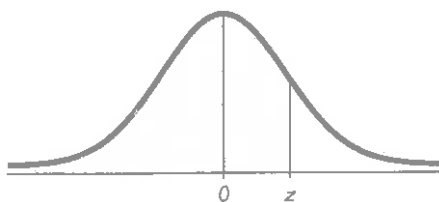
Table A-2 Standard Normal (z) Distribution: Cumulative Area from the LEFT

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.50 and lower	.0001									
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

NOTE: For values of z below -3.49, use 0.0001 for the area.

*Use these common values that result from interpolation:

z score	Area
-1.645	0.0500
-2.575	0.0050



POSITIVE z Scores

Table A-2 (continued) Cumulative Area from the LEFT

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.50 and up	.9999									

NOTE: For values of z above 3.49, use 0.9999 for the area.

*Use these common values that result from interpolation:

z score	Area
1.645	0.9500
2.575	0.9950

Common Critical Values

Confidence Level	Critical Value
0.90	1.645
0.95	1.96
0.99	2.575

Table A-3 *t* Distribution: Critical *t* Values

Degrees of Freedom	0.005	0.01	Area in One Tail 0.025	0.05	0.10
	0.01	0.02	Area in Two Tails 0.05	0.10	0.20
1	63.657	31.821	12.706	6.314	3.078
2	9.925	6.965	4.303	2.920	1.886
3	5.841	4.541	3.182	2.353	1.638
4	4.604	3.747	2.776	2.132	1.533
5	4.032	3.365	2.571	2.015	1.476
6	3.707	3.143	2.447	1.943	1.440
7	3.499	2.998	2.365	1.895	1.415
8	3.355	2.896	2.306	1.860	1.397
9	3.250	2.821	2.262	1.833	1.383
10	3.169	2.764	2.228	1.812	1.372
11	3.106	2.718	2.201	1.796	1.363
12	3.055	2.681	2.179	1.782	1.356
13	3.012	2.650	2.160	1.771	1.350
14	2.977	2.624	2.145	1.761	1.345
15	2.947	2.602	2.131	1.753	1.341
16	2.921	2.583	2.120	1.746	1.337
17	2.898	2.567	2.110	1.740	1.333
18	2.878	2.552	2.101	1.734	1.330
19	2.861	2.539	2.093	1.729	1.328
20	2.845	2.528	2.086	1.725	1.325
21	2.831	2.518	2.080	1.721	1.323
22	2.819	2.508	2.074	1.717	1.321
23	2.807	2.500	2.069	1.714	1.319
24	2.797	2.492	2.064	1.711	1.318
25	2.787	2.485	2.060	1.708	1.316
26	2.779	2.479	2.056	1.706	1.315
27	2.771	2.473	2.052	1.703	1.314
28	2.763	2.467	2.048	1.701	1.313
29	2.756	2.462	2.045	1.699	1.311
30	2.750	2.457	2.042	1.697	1.310
31	2.744	2.453	2.040	1.696	1.309
32	2.738	2.449	2.037	1.694	1.309
33	2.733	2.445	2.035	1.692	1.308
34	2.728	2.441	2.032	1.691	1.307
35	2.724	2.438	2.030	1.690	1.306
36	2.719	2.434	2.028	1.688	1.306
37	2.715	2.431	2.026	1.687	1.305
38	2.712	2.429	2.024	1.686	1.304
39	2.708	2.426	2.023	1.685	1.304
40	2.704	2.423	2.021	1.684	1.303
45	2.690	2.412	2.014	1.679	1.301
50	2.678	2.403	2.009	1.676	1.299
60	2.660	2.390	2.000	1.671	1.296
70	2.648	2.381	1.994	1.667	1.294
80	2.639	2.374	1.990	1.664	1.292
90	2.632	2.368	1.987	1.662	1.291
100	2.626	2.364	1.984	1.660	1.290
200	2.601	2.345	1.972	1.653	1.286
300	2.592	2.339	1.968	1.650	1.284
400	2.588	2.336	1.966	1.649	1.284
500	2.586	2.334	1.965	1.648	1.283
1000	2.581	2.330	1.962	1.646	1.282
2000	2.578	2.328	1.961	1.646	1.282
Large	2.576	2.326	1.960	1.645	1.282

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Table A-4 Chi-Square (χ^2) Distribution

Degrees of Freedom	Area to the Right of the Critical Value									
	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
1	—	—	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.071	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.299
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.042	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.194	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.257	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.954	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169

Source: From Donald B. Owen, *Handbook of Statistical Tables*.

Degrees of Freedom

$n - 1$	Confidence Interval or Hypothesis Test with a standard deviation or variance
$k - 1$	Goodness-of-Fit with k categories
$(r - 1)(c - 1)$	Contingency Table with r rows and c columns
$k - 1$	Kruskal-Wallis test with k samples